Introduction

During my journey from Paris on a TGV train, I was intrigued as to how and why the train was able to brake silently. Through research, I was able to determine that this was because of magnetic damping/braking. I became interested in exploring whether a larger exposed area would have a stronger effect as opposed to a smaller area.

Magnetic damping is a form of drag that occurs when an electrical conductor travels past a magnetic field. This process concerning a pendulum with a copper bob will see the bob's motion become dampened as it exits and enters the magnetic field produced by two magnets. This document strives to compose and propose the correlation between the area of a copper oscillator how that affects the rate of change of amplitudes of the oscillations. Thus, one arrives at the following research question:

How does varying the area of a copper oscillator affect the rate of change of amplitude due to magnetic damping? RQ clear and focused

Theory

Lenz's Law states that the induced current always flows in the direction that opposes the change in magnetic flux.

The said motion induces an eddy current in the conductor as a magnetic field is present. As per Lenz's law the movement of electrons in the conductor instantly produce an opposing magnetic field which results in the damping of the oscillation ultimately 'slowing' down the metal bob.

When the metal plate is completely inside the field thus at rest, there is no eddy current. When the plate leaves the field on the right, it causes an eddy current in the clockwise direction that, again, experiences a force to the left, further slowing the bobs motion.[1] This produces a exponentially damped harmonic which can be expressed with the following equation:

$$x(\mathrm{t}) = \mathrm{A}e^{-\mathrm{bt}}cos(\omega'\mathrm{t} + ø) \qquad [1]$$ [2]

Where:
x is the displacement
t is the time of release
A is the amplitude
ω′ is the angular frequency of damped simple harmonic motion.
b is the damping coefficient
ø is the Angular frequency

[1] https://courses.lumenlearn"Eddy Currents and Magnetic Damping | Physics." Lumenlearning.Com, 2020, courses.lumenlearning.com/physics/chapter/23-4-eddy-currents-and-magnetic-damping/. Accessed 6 Mar. 2020
[2] "Toppr." Toppr-Guides, 27 Feb. 2018, www.toppr.com/guides/physics/oscillations/damped-simple-harmonic-motion/. Accessed 6 Mar. 2020.

This equation and information attained can then be applied in order to evaluate the following factors:

The logarithmic decrement is defined as the natural log of the ratio of the amplitudes of any two successive peaks: (The logarithmic decrement represents the rate at which the amplitude of a damped harmonic decreases.[3])

$$\varphi = ln\left(\frac{A_i}{A_{i+1}}\right) \quad [2]$$ [4]

Where:
A_i is a peak amplitude
A_{i+1} is the following peak amplitude

The damping ratio is then found from the logarithmic decrement by: (The damping ratio is a measure that expresses the decay of oscillations)

$$\zeta = \frac{\varphi}{\sqrt{(2\pi)^2 + \varphi^2}} \quad [3]$$ [5]

Where:
φ is the logarithmic decrement
ζ is the damping ratio.

The experiment consists of 8 different oscillator areas (and one control with no copper plate) that will be oscillating on a pivot. The displacement of the oscillator will be recorded using a camera. Then, video analysis will be used to extract data, that will be processed to find the peak amplitudes which are integral to the aforementioned equations.

Background info relevant to the following actions

Design of Experiment

The design below is the front and side view of the setup that was created for this investigation. How this apparatus is used and controlled is discussed in later sections.

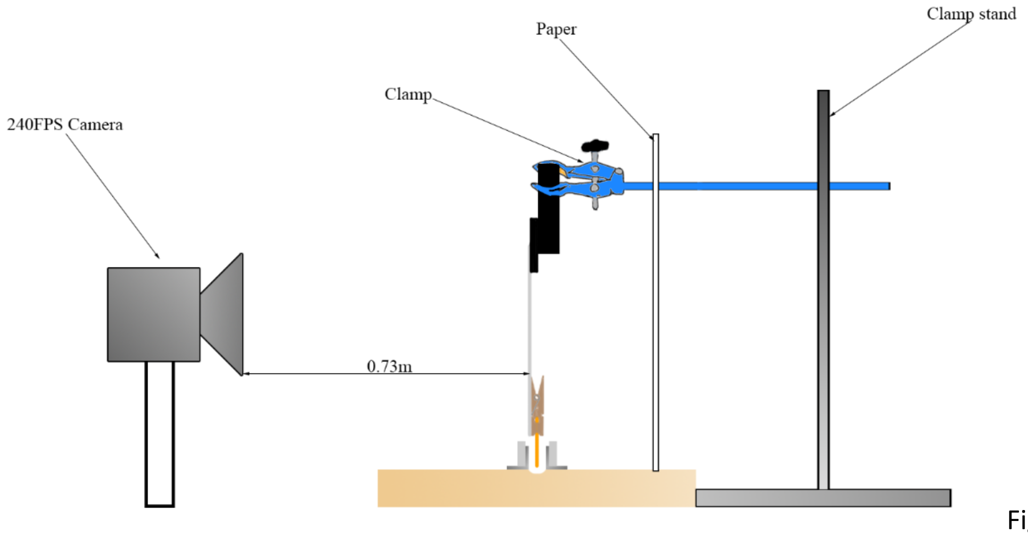


Figure 1 , experiment setup

[4] Ibid
[5] Ibid

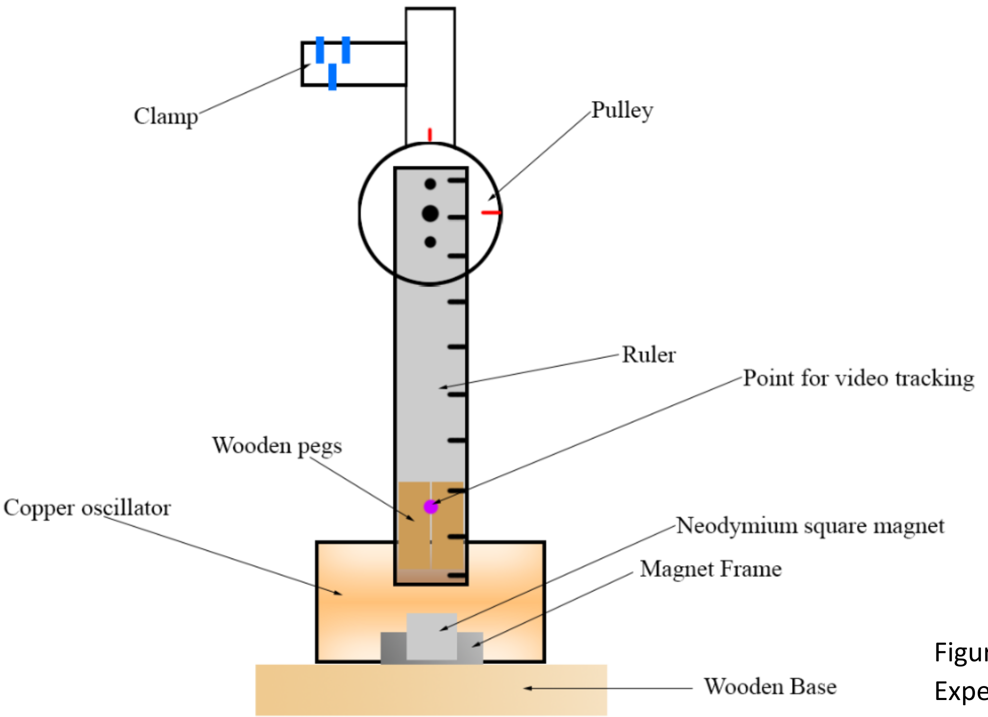


Figure 2,
Experiment setup

The variables for the experiment can be seen in the table below:
Table 1: Variables for explanation

Independent variable	Area of the copper oscillator
How was it measured?	With a ruler. Limit of reading?
Dependent variable	Rate of change of amplitude
How was it measured?	Video analysis. and then what?

Table 2: Control Variables

eV detailed and rigorous procedure to ensure quality data

Controlled variable	Why significant?	How is it controlled practically?
The starting angle of the pendulum is perpendicular in every run. ry nice,	To have a controlled experiment, the angle from which the oscillator will start oscillating is significant as the initial position of the marked point would be different in each run therefore hindering the integrity of the data collected. The initial deflection will also be my first data point. (**16** cm)	To ensure that the oscillation starts at a 90° angle, a small marking is drawn. The markings are made to match before the oscillator is released. The angle was checked using a spirit level. As seen in Figure 2.
Height used for the oscillator of plate.	Since I would be changing area, I made sure that the height stayed the same each time and I would only be cutting the width of **1.5** cm each time starting from the maximum width of **20** cm. The controlled height was **5.2**cm.	Engravings were made on the sheet of metal; this was done to make the area cutting method more comfortable. It was made sure that the height stays uniform.

Table 2: Experimental precautions

Experimental precautions	**Why significant?**	**How is it controlled practically?**
The camera is the same distance from the pendulum between takes.	Since the use of a camera has the effect of a parallax error. To maintain consistency, the arrangement and location of the camera must be the same throughout.	I set down a meter stick and made sure that the camera 'stand' was **0.73m** away from the oscillating body using post-it notes to ensure the location of the phone holder is the same.
The magnets do not move location or angle.	In order to have a fair experiment the separation between the magnets and the copper oscillator must be the same, so that the magnetic field will be uniform throughout all runs of the experiment. I made a mark of their location, so it would be very easy for me to check if the location stayed consistent.	Using a set square, I was able to mark where the wood frame should be located. Also, the magnet holding slits are screwed down into the wooden platform to ensure the uniform separation between the two magnets.
The same frame rate will be used for each run.	If the frame rate differs in the videos, then the integrity of the results will not be upheld. As when the data is being collected through video analysis, the time intervals for each frame would be different.	The video is taken every time at 240 FPS, then it is refactored using ***ffmpeg*** into 120FPS for acceleration of the data collection process.
Same location for the clips that hold the copper plate.	The same height of the copper oscillator should be exposed to the magnetic field. To experience the same force/damping.	A line is drawn onto the sheet of copper to ensure that the ends of the pegs have the same alignment for each run/area.

Area selection: The results had to vary enough to state and evaluate whether a correlation is present between area and magnetic damping. This meant that the area that was used could not be too large as then the results would not vary. Therefore, many trail runs were conducted to determine the fit range of areas. I found that areas that exceeded 100cm^2 would not oscillate at all. The wooden base did not allow for plates that exceeded 80cm^2.

Tracking point location: The tracking point location was selected, as the magnets would hide the point if it were to be placed at the bottom of the oscillator.

Table 3: Oscillator area table

Area of oscillators			
Height ±0.05 (cm)	Width ±0.05 (cm)	Area (cm^2)	ΔArea (cm^2)
5.2	12.0	62.4	0.9
5.2	10.5	54.6	0.8
5.2	9.00	46.8	0.7
5.2	7.50	39.0	0.6
5.2	6.00	31.2	0.6
5.2	4.50	23.4	0.5
5.2	3.00	15.6	0.4
5.2	1.50	7.80	0.3

The table above is of the areas of the oscillators that are used for the experiment. The height is controlled at 5.2 ±0.05 cm.

The uncertainty for area was found using the following equation:

$$\Delta Area = \frac{\big((H+0.05)\times(W+0.05)\big)-\big((H-0.05)\times(W-0.05)\big)}{2} \quad [4]$$

C: Correct data processing

Data Collection:

First, due to the time constraints, the frame rate of the video had to be changed. Therefore, each one of the videos was refactored from 240FPS to 120FPS. This was done using a free command-based video and audio manipulating library called ffmpeg[6]. This process was done using the following command:

```
ffmpeg -i 'input file path' -filter:v "setpts=PTS/2" 'output file path' 3
```

The videos were then analysed using the free program *Tracker* [7]. For each video, I set the axis (0,0) at where the **tracking point** was at rest. This is because to graph the oscillation, the displacement in x is needed. Then the boundaries of the video were set, where the starting frame would be the frame that the body began to oscillate. Then the video had to be calibrated to counteract the parallax that was caused by the camera. This was done with the inbuilt filter called '*perspective*'. Then to allow the program to understand distances that are represented in the video, the '*calibration tape*' tool was used to set the calibrate distances within the video. And the scale was set from the pivot point to the tracking point and was measured as **16cm.**

A: New software learnt

[6] "Ffmpeg Documentation." Ffmpeg.Org, 2020, ffmpeg.org/ffmpeg.html. Accessed 11 Feb. 2020.
[7] "Tracker Video Analysis and Modeling Tool for Physics Education." Physlets.Org, 2019, physlets.org/tracker/. Accessed 14 Feb. 2020.

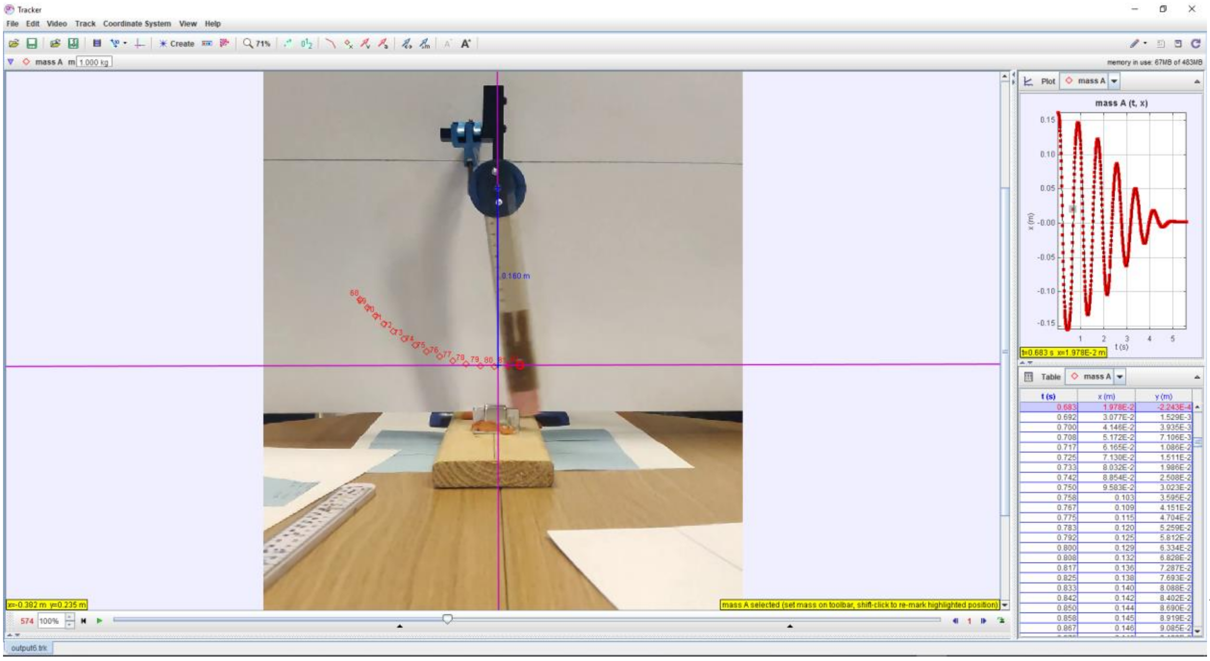


Figure 3, Tracker software

Then for the tracking procedure another inbuilt feature was used to cut down on the vigorous process of hand tracking. This feature is called '*Auto tracker*' and as the name suggests this a process where the software itself uses an image matching algorithm to scrub and track a selected point. This was based on the concept of the user setting a template for a point, and the software would find and plot a point that matches set template and then increments the frames until the last point is plotted. As shown in Figure 4.

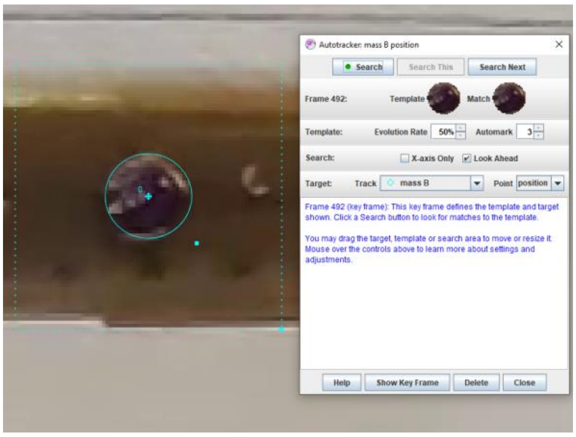


Figure 4, Auto-tracking feature

This process was continued for all nine of the areas, and the following table is a snippet of the output.

Table 4: Raw data sample table

You should have stated that the raw data is in the appendix.

area cm^2	
time(s)	x(m) ±0.005
0.00E+00	1.60E-01
8.33E-03	1.60E-01
1.67E-02	1.60E-01

The graph below is the displacement placed against time for each one of the areas, the damping in relation to the area used is clear as seen the varying time it takes for the systems to become at rest.

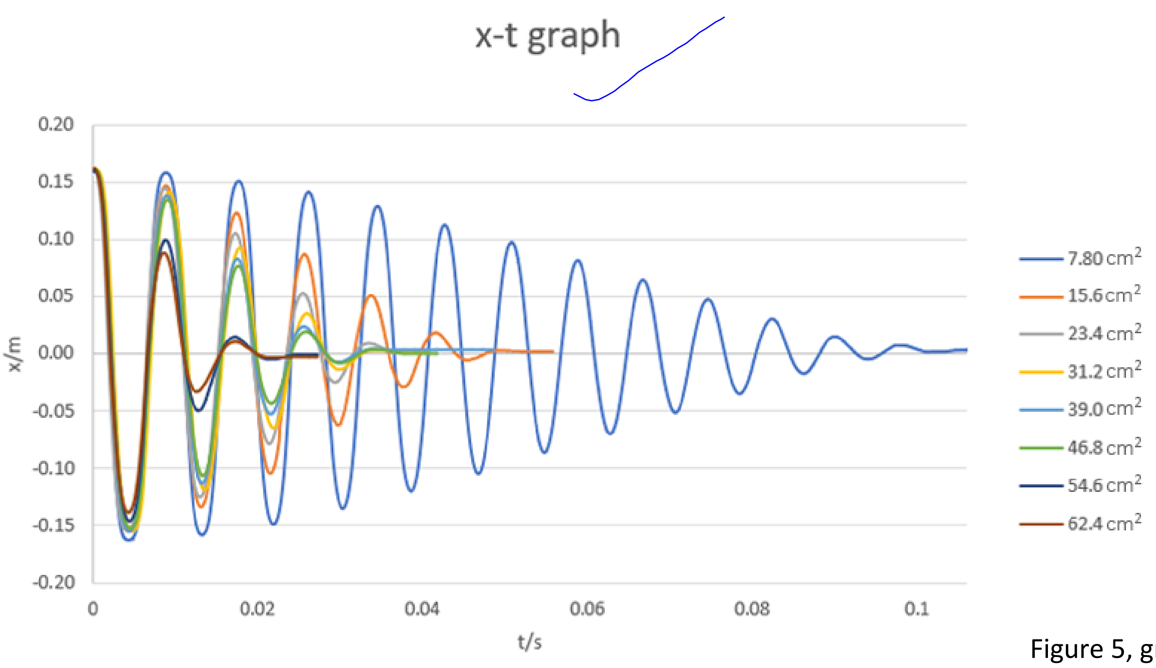
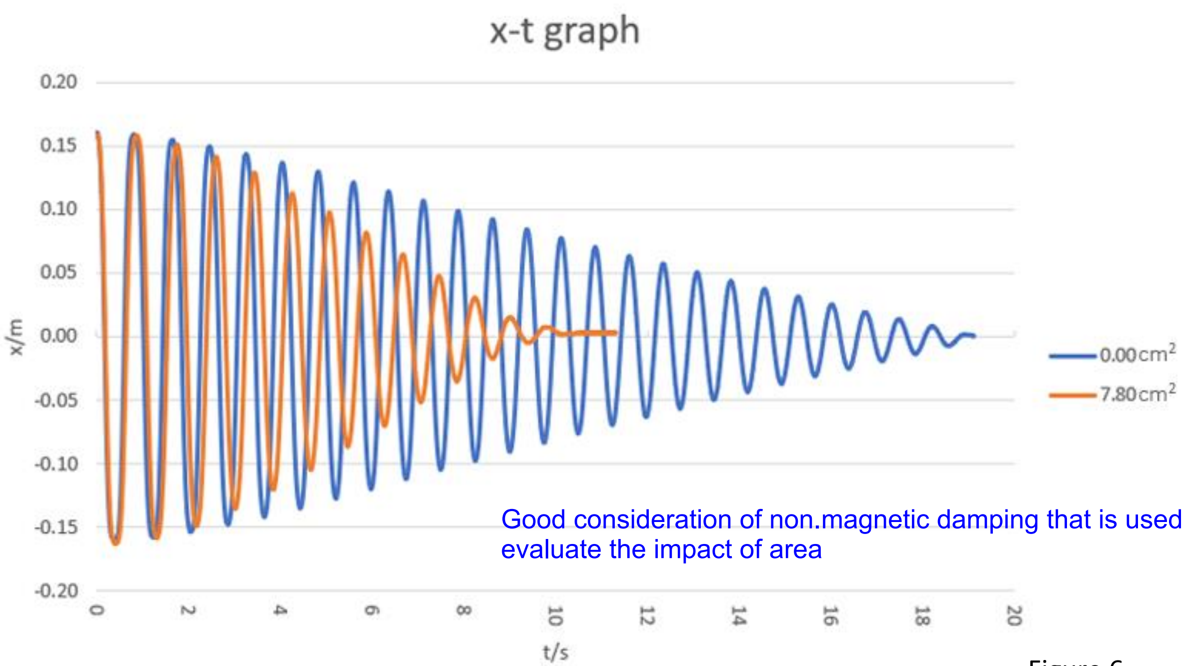


Figure 5, graphed data

The graph below the same visual as below however the two data sets shown are of the undamped system and the smallest area, where the no matter how small still makes a significant difference in the damping.



Good consideration of non.magnetic damping that is used later to evaluate the impact of area

Figure 6, Graphed data

Data Processing:

Due to the inherent implications of magnetic damping, the data attained will be used to evaluate the aforementioned concepts that were discussed in the Theory section.

As suggested by the formula for **Logarithmic Decrement** (φ), in order to evaluate the damping, the ratio of two successive peaks must be attained. This was done using *MagicPlot*[8] software, due to its ease of use and simple extraction of selected data points.

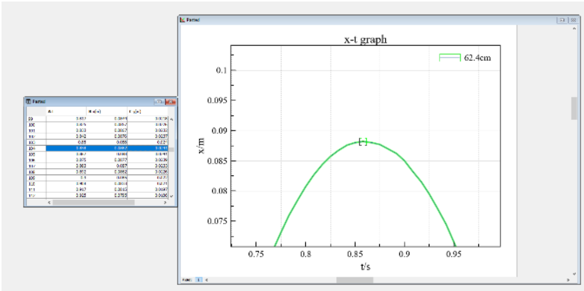


Figure 7, Magic plot

These peaks were then noted down with their corresponding times which are needed for further analysis.

Table 4: Sample processed data table (15.6 cm^2):

Time(s)	Distance in x of peak(m) ±0.005	Ratio	ΔRatio	Logarithmic decrement	ΔLogarithmic decrement	Damping factor	ΔDamping Factor
0.00	0.160	1.14	0.02	0.134	0.019	0.0300	0.0000
0.89	0.140	1.30	0.02	0.260	0.036	0.0583	0.0002
1.74	0.108	1.29	0.03	0.251	0.040	0.0565	0.0002
2.57	0.084	1.66	0.05	0.509	0.108	0.114	0.001
3.37	0.051	2.79	0.21	1.03	0.361	0.225	0.007
4.16	0.018	3.62	0.96	1.29	0.658	0.278	0.015
5.02	0.005						

consistent s.f.

COnsistent d.p.

- The Time and coordinate of peaks were imported from *MagicPlot*.
- The ratio was then attained by dividing one peak coordinate by its successor.
- The relative uncertainty for the ratio was attained as such:

$$\Delta Ratio = \frac{\left(\frac{x_1 + \Delta x_1}{x_2 - \Delta x_2}\right) - \left(\frac{x_1 - \Delta x_1}{x_2 + \Delta x_2}\right)}{2} \quad [5]$$

correct error processing

Where:

x_1 is the set peak, x_2 is the following peak, and the equation represents the smallest minus the largest difference.

- The uncertainty for the Logarithmic Decrement was attained as such:

$$\Delta\varphi = \frac{ln\left(\frac{x_1 + \Delta x_1}{x_2 - \Delta x_2}\right) - ln\left(\frac{x_1 - \Delta x_1}{x_2 + \Delta x_2}\right)}{2} \quad [6]$$

- The uncertainty for the damping factor was attained as such:

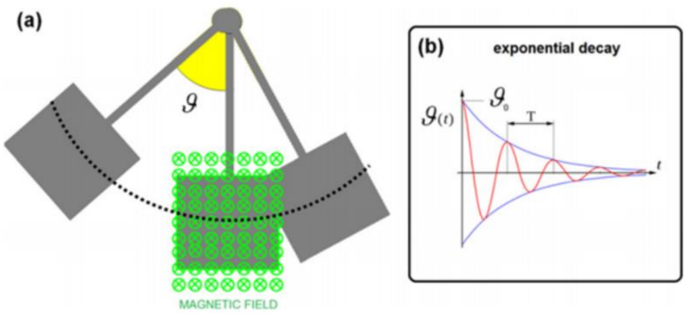
$$\Delta\zeta = \zeta\left(\frac{2\Delta\varphi}{\varphi}\right) \quad [7]$$

There is a variety of error calulations, all relevant to the techniques used.

The remainder of the processed data can be found in the appendix.

Analysis

The principal effect of damping is to reduce the amplitude of an oscillation, not to change its frequency. So, the graph of the amplitude of a normal damped oscillation would look like the following:



(a) A metallic pendulum oscillates inside and outside the magnetic field region generated by a permanent magnet. (b) The oscillation amplitude decays exponentially with time as described by Eq. [8]

Figure 8 : SHM damping decay

This means that a graph of all my peaks should too follow a pattern of an exponential curve, this however is not the case. As when all the peaks are graphed for each area, they follow a linear pattern as shown in Figure 9 rather than one of exponential decay shown in Figure 8. This is due to the inherent systematic error that was prominent throughout the data collection process which will be commented and explained further in the evaluation section.
This is a model based on statistical patterns of the data, but the explanation would require a very complex mathematics since the magnetic field of the magnets is not uniform. (A systematic error that is explored in the Evaluation).

Good critical thinking displayed

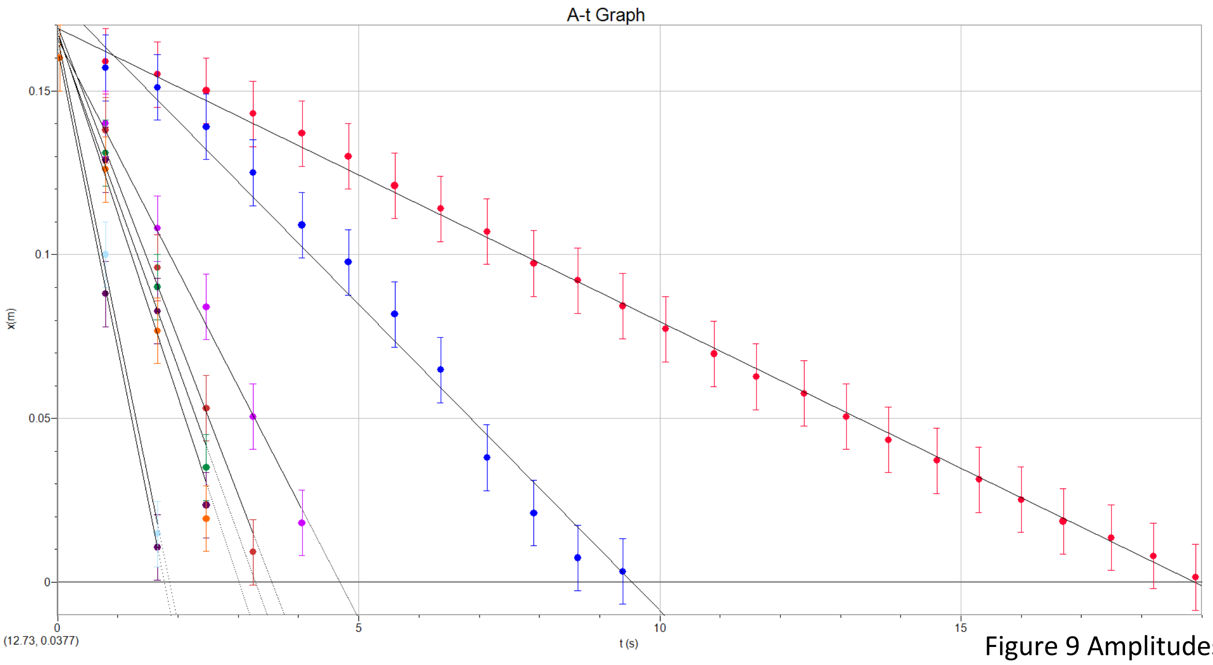


Figure 9 Amplitudes over time graph

[8] P. Onorato, and Anna De Ambrosis. "Magnetic Damping: Integrating Experimental and Theoretical Analysis." ResearchGate, 21 Dec. 2011, Accessed 5 Feb. 2020.

This inherent issue with my experiment is that it does not allow me to evaluate the pattern using a conventional method, which was outlined in the theory section. This is due to the general linearity of my data, which instantaneously dismisses the processed data in Table 4, as the methods discussed apply to systems that stop oscillating exponentially. This meant that I had to find a way to explore the relationship in an unconventional method. Also, due to the nature of the data, It is impossible to evaluate the Q (Quality) factor, as the data does not meet the necesary requirements. Engineers use the Q factor to evaluate the dimensionless parameter that describes how underdamped an oscillator is, which can also be applied to magnetic damping.

Nice. Data does not follow theoretical pattern, so there is a new method to quantify the damping.

The gradients for each of these are attained inorder to evaluate how each area affected the oscillation. I attained the error by establishing error bars then attaining the maximum, minimum and average gradients as shown by Figure 10 which is an example of a 64.2cm^2 area.

This process was repeated for each one of the areas, as it will be needed for later evaluation of the errors in search of assessing the relationship between area and damping.(See appendix)

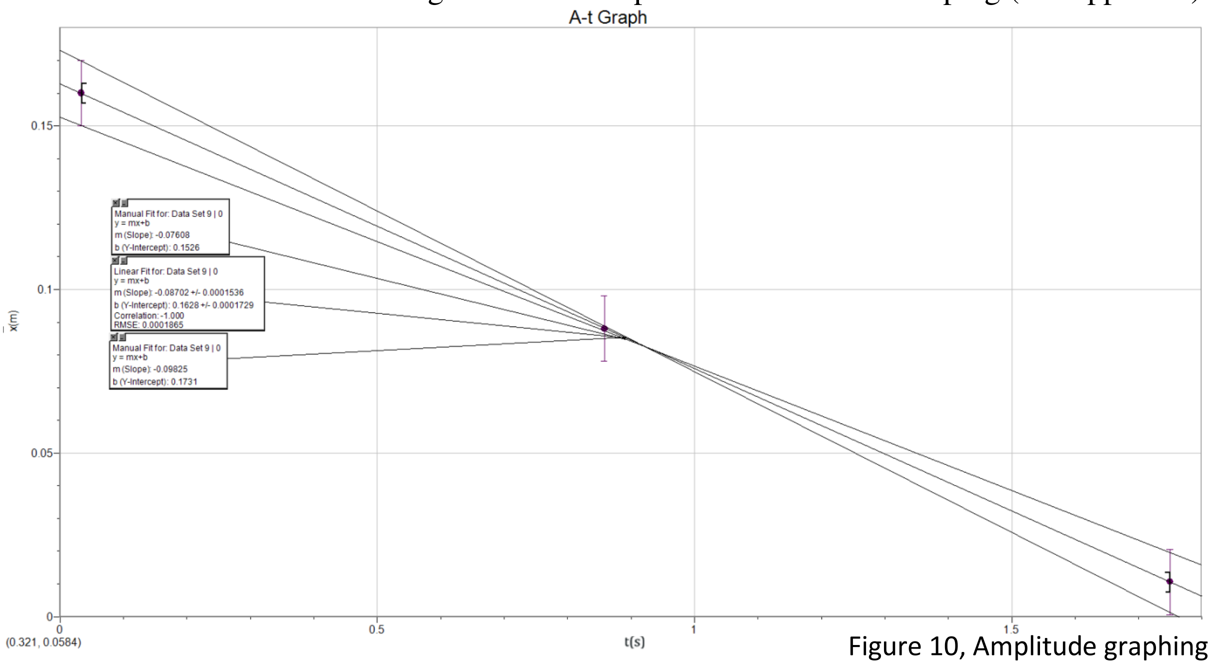


Figure 10, Amplitude graphing example for which area is it?

The graidnets attained are shown by table 5:

	Area (cm^2)	Grad / BEST (ms^{-1})	Grad / MIN (ms^{-1})	Grad / MAX (ms^{-1})	ΔGrad (ms^{-1})
0	0.00	-0.008955	-0.007862	-0.009743	0.00094
1	7.80	-0.01672	-0.01560	-0.01788	0.00114
2	15.6	-0.03463	-0.03145	-0.03782	0.00319
3	23.4	-0.04646	-0.04402	-0.04965	0.00282
4	31.2	-0.04867	-0.04577	-0.0514	0.00282
5	39.0	-0.05399	-0.05584	-0.05706	0.00061
6	46.8	-0.05871	-0.05188	-0.05534	0.00173
7	54.6	-0.08609	-0.07955	-0.09283	0.00664
8	64.2	-0.08702	-0.07608	-0.09825	0.01109

Again, another error analysis, this time for the graidents.

The ΔGradient was attained using the following equation:

$$\Delta Grad = \frac{Grad_{MAX} - Grad_{MIN}}{2} \qquad [8]$$

This table allows us to then graph the different gradient with their accompanying error bars, As shown in figure 11. While we are not able to fit any one regression, the relationship is shown by the data points. We are able to see the relationship that the area has on the time it takes for an oscillating system to stop due to magnetic damping. Gradient is put against Area. It can be seen that with the larger areas the steeper the gradient is (As signified by the decreasing values). This means that the larger the area of the copper oscillator is, the faster it will come to a halt.

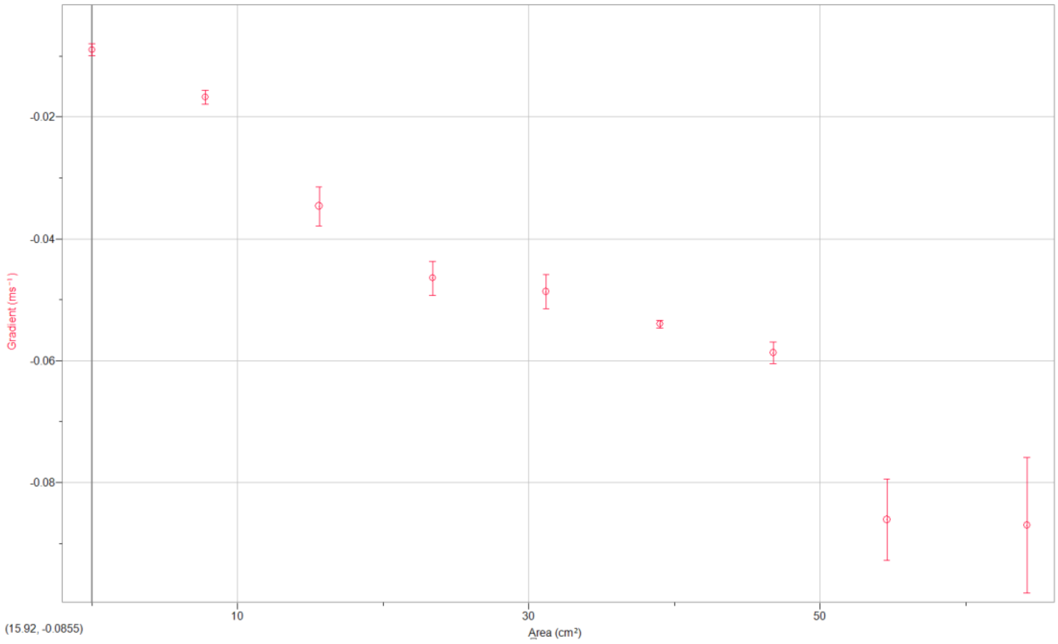


Figure 11 Gradient vs Area

Conclusion:

This internal assessment has successfully underlined and explored how the area of a copper oscillator affects the rate of damping experienced by an oscillating pendulum. It can be concluded that with a higher area that is exposed to a magnetic field, the stronger the damping effect will be. The applied magnetic field and the copper sheet produce eddy currents that are a force that opposes the motion of the bob. This is due to the internal resistance of the conductor, the said eddy currents will dissipate into heat, causing a removal of energy from the system. This dissipation of energy allows the magnet and conductor to form a damper that may be used to suppress the oscillation [9].

SOme explanation

As with the larger area, more eddy currents can be induced across, meaning a larger force is created. Some of the damping is due to the friction of the pully. The experiment has a lot of sources of uncertainty.

D: To gain full marks, more interpretations of the last graph are expected.

The experiment, however, was not perfect and therefore it did not yield data that follow the Logarithmic decrement requirements and thus yielding results that could not be used and evaluated using a standard method. This motivated me to propose an alternative and unique method to present and evaluate the relationship. The corresponding gradients of the

[9] Sodano, Henry, et al. Improved Eddy Current Damping Model for Transverse Vibrations. , Accessed 5 Feb. 2020.

amplitudes were graphed against area to determine whether a relationship exists. The graph suggested that with the larger the area, more energy was dissipated, which brought the system to a stop oscillating at a quicker pace. Thus, enforcing the presence of a relationship.

Evaluation:

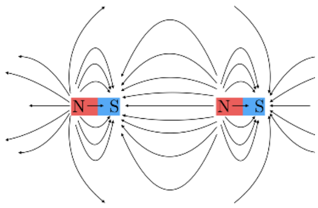
Experimental issues and suggested improvements:

Inactive damping force:

In an ideal linear damped oscillating system, the damping force is always "active" and is proportional to velocity. That's not the case for this setup because the damping happens when the plate is passing between the magnets at other times, when it is at the end of its swing and far away from the magnets, the only damping is from friction. When the amplitude of the swing gets smaller, the plate spends more of its time close to the magnets so the effective amount of damping increases. Therefore, you would expect the initial rate of damping to be small for the amplitude decreases, and the plate is affected more.

Non-uniformity of magnets:

Another source of nonideality comes from the fact that the magnetic field produced by a real magnet is not uniform, So the damping is strongest near the centre of the magnet where the field is strongest. As shown by figure 12. The larger amount of lines means that in the direct centre between of magnets, would result in a stronger effect.



[10]

Figure 12, Magnetic field lines

Further sources of nonideality:

Mechanical friction of the pully caused the oscillator to dissipate energy. Another source of nonideality would be the random error caused by the auto tracking software, as the points would start deviating over time, if it was done manually it could potentially improve accuracy.

Improvements:

A number of improvements can be made to fix the aforementioned errors:

- Conduct trails to find a suitable starting angle to make sure oscillator is always affected by the magnetic field. Such as a smaller angle and a larger field area.
- Rather than use displacement as a way of data collection, the use of the angle could more adhere to the equation for damping.
- As most of the data collection was done using a computer algorithm, errors in plotting may have occurred. Thus, many runs of the data collection/analysis could be done and then the average of the runs would be used.
- To reduce friction, a pully with lubricated bearings would be used.

[10] "Magnetic Field Lines | Brilliant Math & Science Wiki." Brilliant.Org, 2020, brilliant.org/wiki/magnetic-field-lines/. Accessed 6 Mar. 2020.

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./Toppr." Toppr-Guides, 27 Feb. 2018, www.toppr.com/guides/physics/oscillations/damped-simple-harmonic-motion" .Accessed 6 Mar. 2020

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"Tracker Video Analysis and Modeling Tool for Physics Education." Physlets.Org, 2019, physlets.org/tracker/. Accesse d 14 .Feb. 2020

P. Onorato, and Anna De Ambrosis. "Magnetic Damping: Integrating Experimental and Theoretical Analysis." ResearchGate, 21 Dec. 2011, Accessed 5 Feb. 2020.

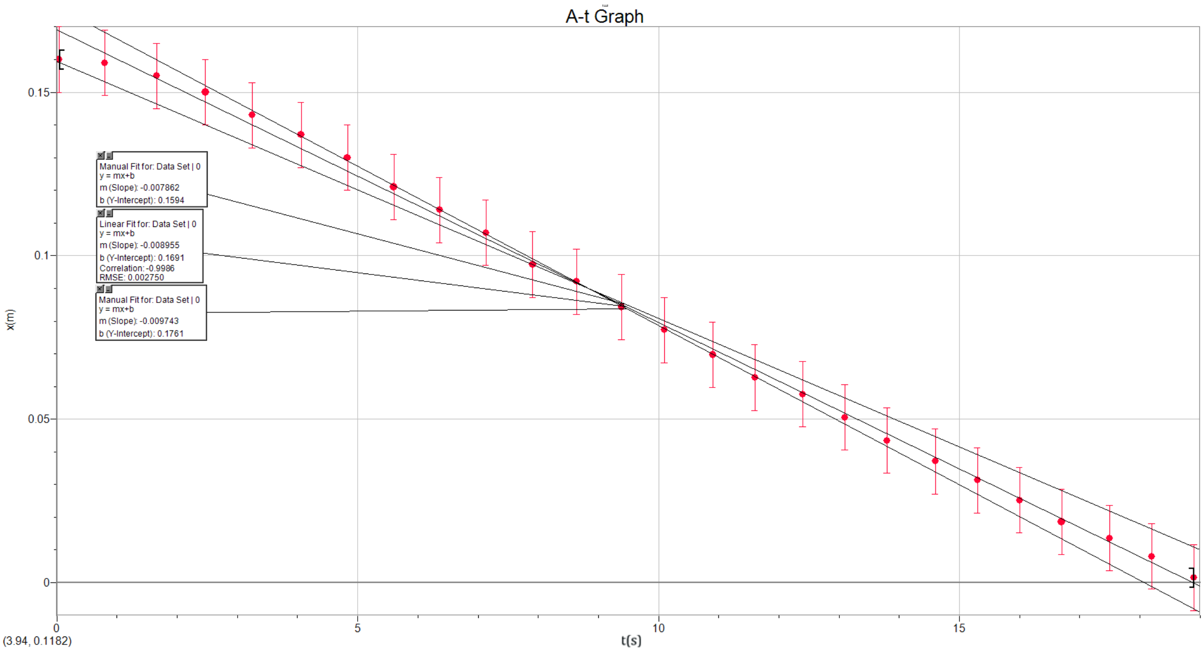
Sodano, Henry, et al. Improved Eddy Current Damping Model for Transverse Vibrations. , Accessed 5 Feb. 2020.

"Magnetic Field Lines | Brilliant Math & Science Wiki ./Brilliant.Org, 2020, brilliant.org/wiki/magnetic-field-lines ". .Accessed 6 Mar. 2020

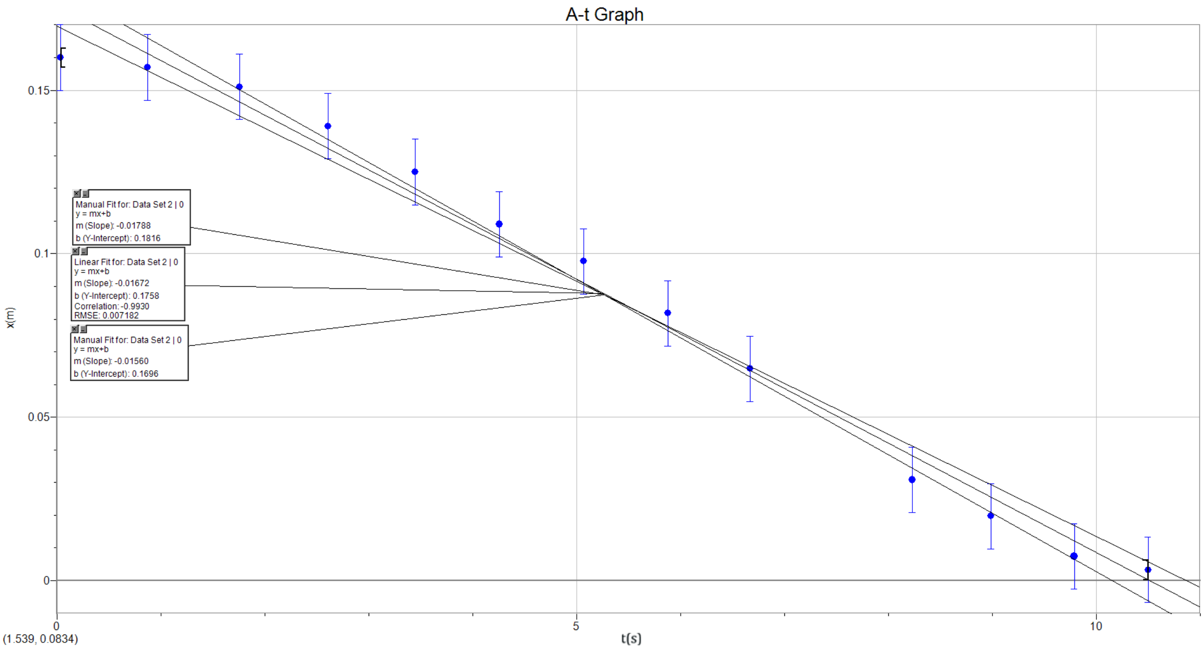
Appendix

1 : Amplitude of areas graphed. (in cm^2)

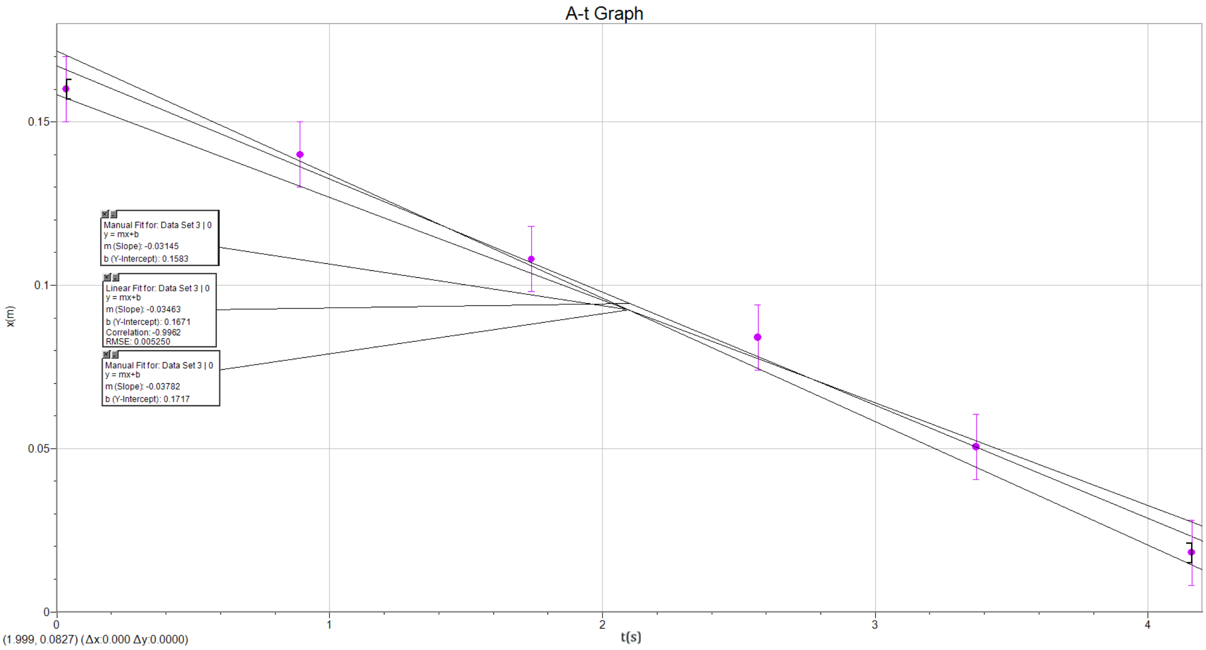
0



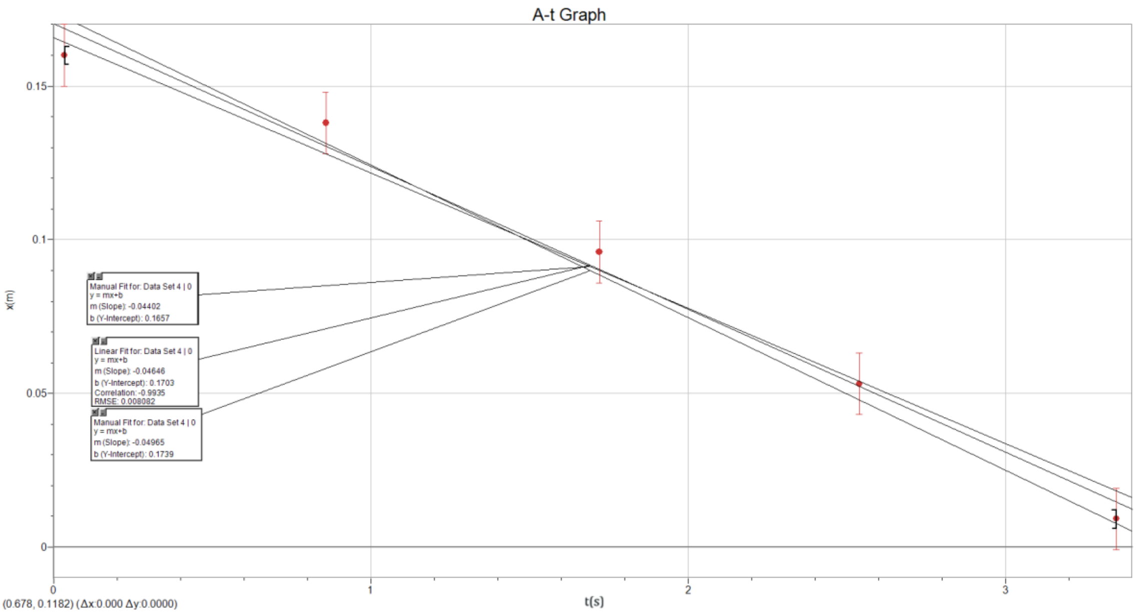
7.8cm



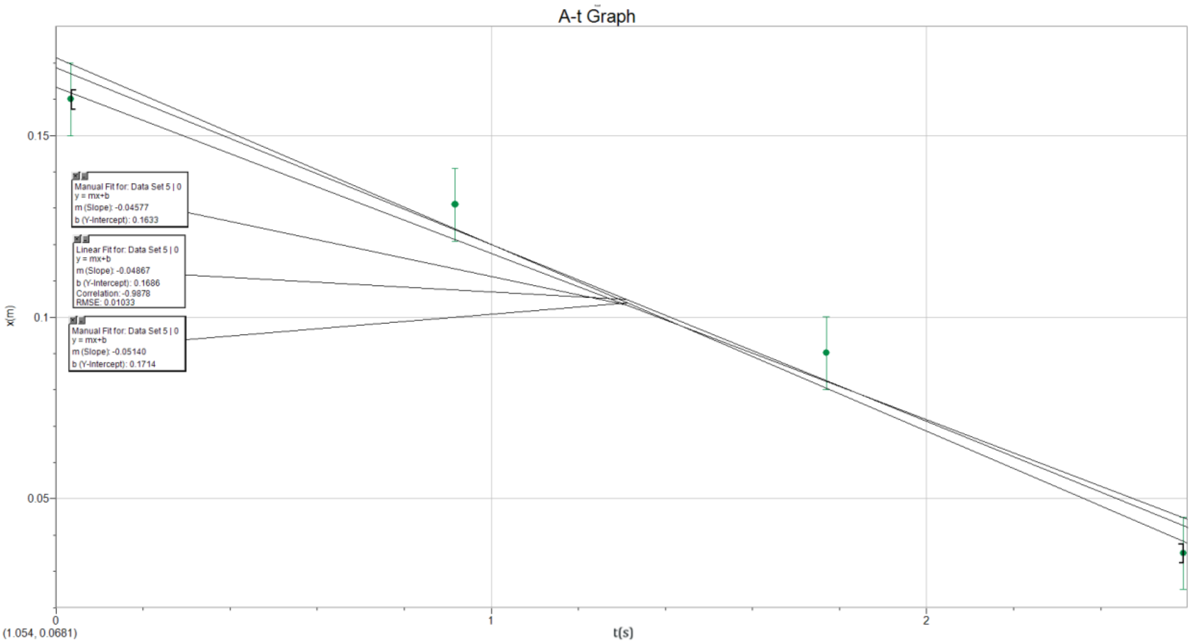
15.6


A-t Graph
Manual Fit for: Data Set 3 | 0
y = mx+b
m (Slope): -0.03145
b (Y-Intercept): 0.1583
Linear Fit for: Data Set 3 | 0
y = mx+b
m (Slope): -0.03463
b (Y-Intercept): 0.1671
Correlation: -0.9962
RMSE: 0.005250
Manual Fit for: Data Set 3 | 0
y = mx+b
m (Slope): -0.03782
b (Y-Intercept): 0.1717
x(m)
t(s)
(1.999, 0.0827) (Δx:0.000 Δy:0.0000)

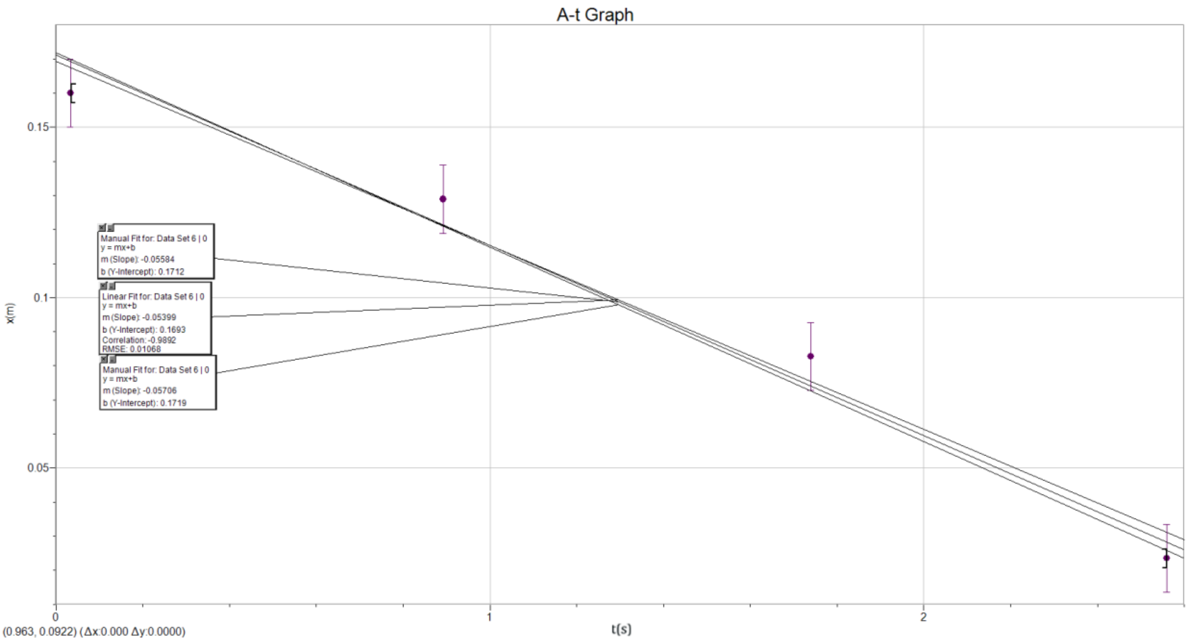
23.4


A-t Graph
Manual Fit for: Data Set 4 | 0
y = mx+b
m (Slope): -0.04402
b (Y-Intercept): 0.1657
Linear Fit for: Data Set 4 | 0
y = mx+b
m (Slope): -0.04646
b (Y-Intercept): 0.1703
Correlation: -0.9935
RMSE: 0.008082
Manual Fit for: Data Set 4 | 0
y = mx+b
m (Slope): -0.04965
b (Y-Intercept): 0.1739
x(m)
t(s)
(0.678, 0.1182) (Δx:0.000 Δy:0.0000)

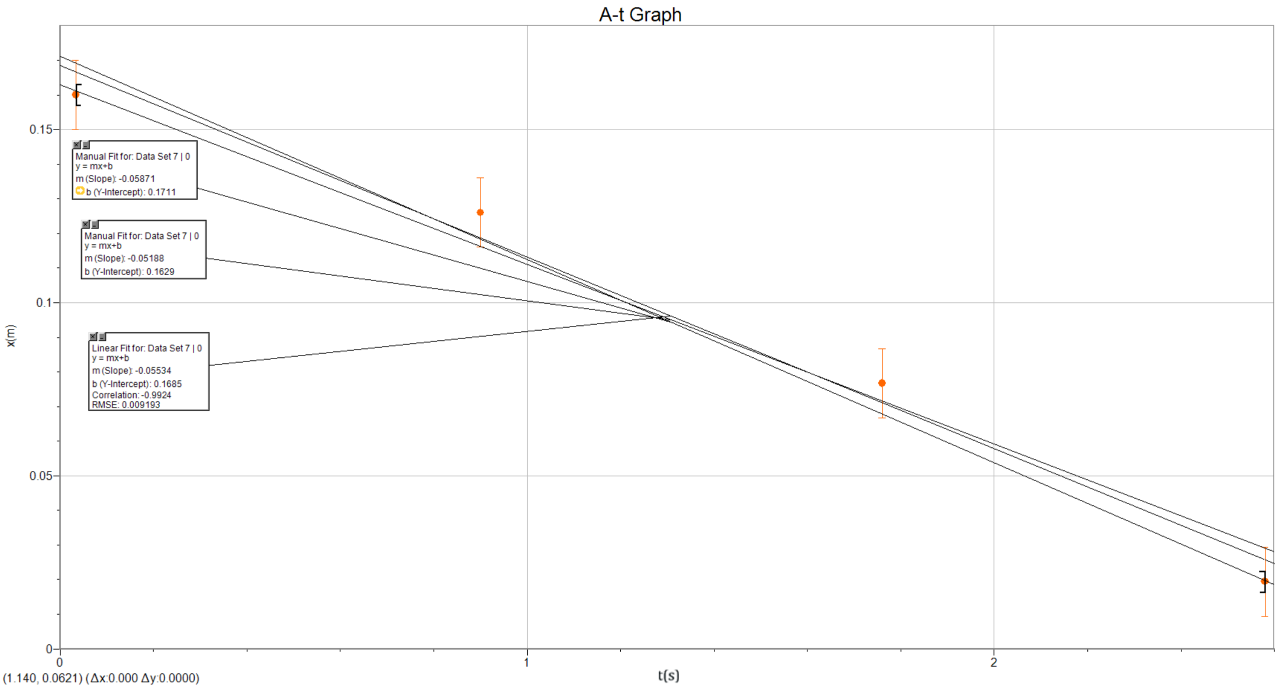
31.2


A-t Graph
Manual Fit for: Data Set 5 | 0
y = mx+b
m (Slope): -0.04577
b (Y-Intercept): 0.1633
Linear Fit for: Data Set 5 | 0
y = mx+b
m (Slope): -0.04867
b (Y-Intercept): 0.1686
Correlation: -0.9878
RMSE: 0.01033
Manual Fit for: Data Set 5 | 0
y = mx+b
m (Slope): -0.05140
b (Y-Intercept): 0.1714
x(m)
t(s)
(1.054, 0.0681)

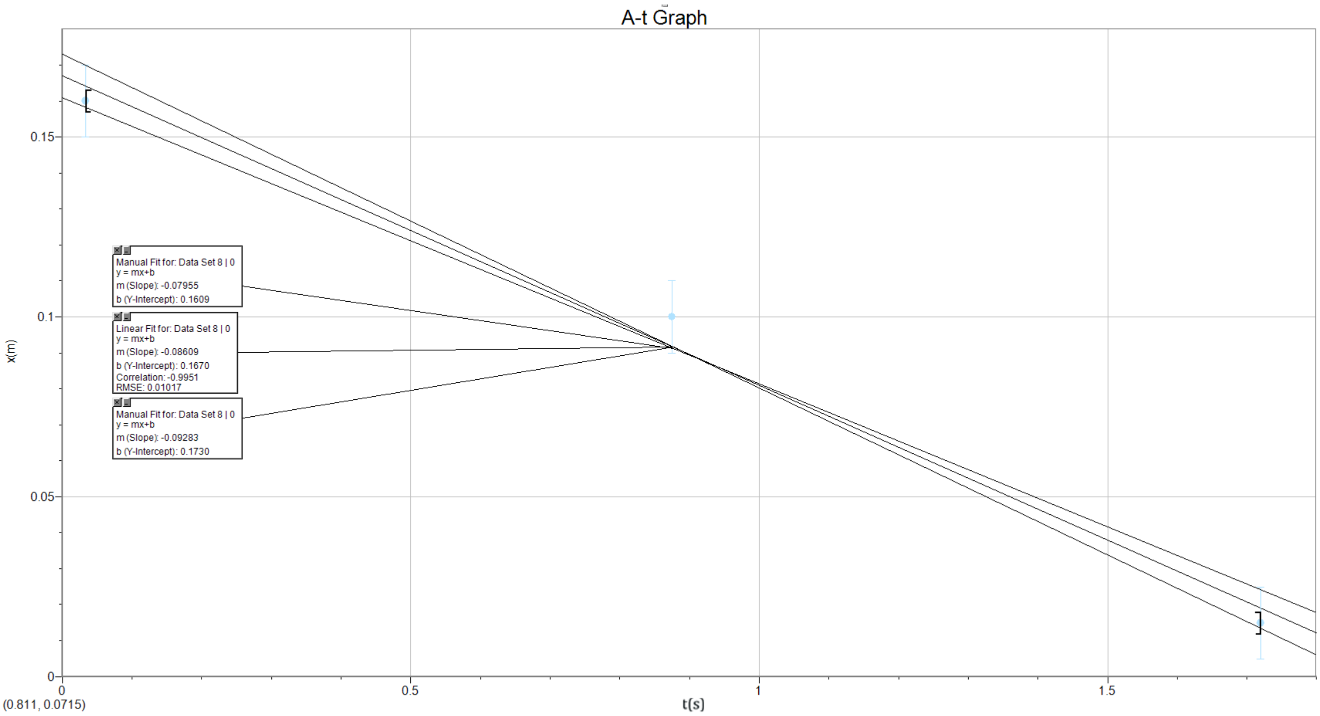
39


A-t Graph
Manual Fit for: Data Set 6 | 0
y = mx+b
m (Slope): -0.05584
b (Y-Intercept): 0.1712
Linear Fit for: Data Set 6 | 0
y = mx+b
m (Slope): -0.05399
b (Y-Intercept): 0.1693
Correlation: -0.9892
RMSE: 0.01068
Manual Fit for: Data Set 6 | 0
y = mx+b
m (Slope): -0.05706
b (Y-Intercept): 0.1719
x(m)
t(s)
(0.963, 0.0922) (Δx:0.000 Δy:0.0000)

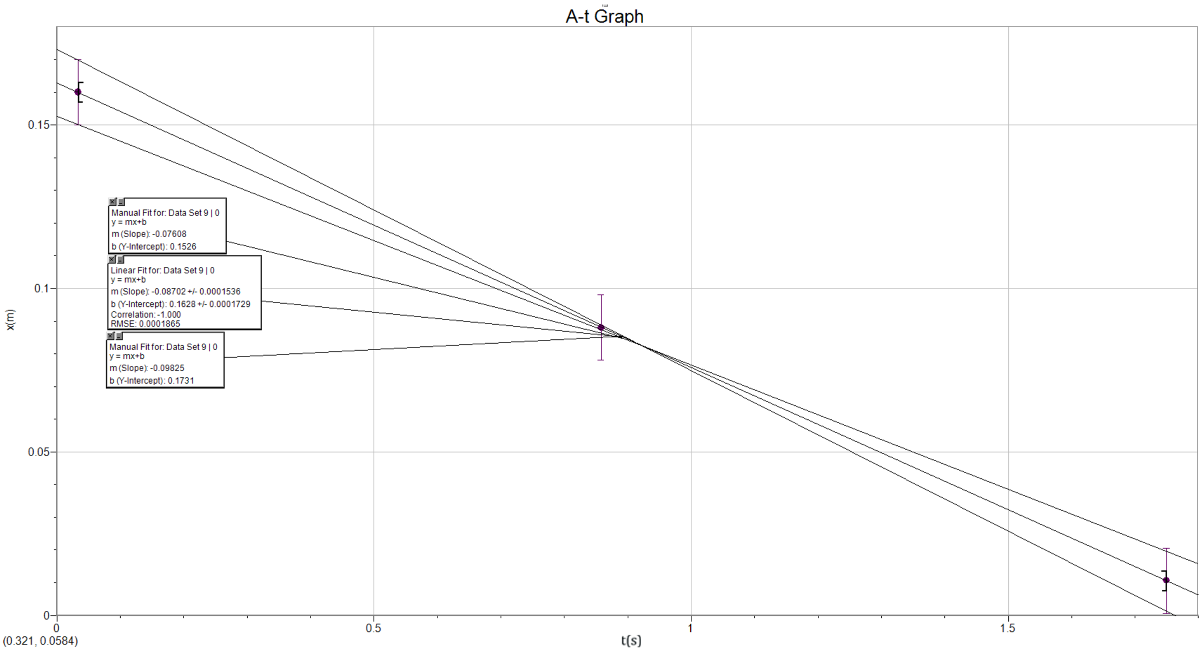
46.8


A-t Graph
Manual Fit for: Data Set 7 | 0
y = mx+b
m (Slope): -0.05871
b (Y-Intercept): 0.1711
Manual Fit for: Data Set 7 | 0
y = mx+b
m (Slope): -0.05188
b (Y-Intercept): 0.1629
Linear Fit for: Data Set 7 | 0
y = mx+b
m (Slope): -0.05534
b (Y-Intercept): 0.1685
Correlation: -0.9924
RMSE: 0.009193
x(m)
0.15
0.1
0.05
0
1
2
t(s)
(1.140, 0.0621) (Δx:0.000 Δy:0.0000)

54.6


A-t Graph
Manual Fit for: Data Set 8 | 0
y = mx+b
m (Slope): -0.07955
b (Y-Intercept): 0.1609
Linear Fit for: Data Set 8 | 0
y = mx+b
m (Slope): -0.08609
b (Y-Intercept): 0.1670
Correlation: -0.9951
RMSE: 0.01017
Manual Fit for: Data Set 8 | 0
y = mx+b
m (Slope): -0.09283
b (Y-Intercept): 0.1730
x(m)
0.15
0.1
0.05
0
0.5
1
1.5
t(s)
(0.811, 0.0715)

62.4



A-t Graph
Manual Fit for: Data Set 9 | 0
y = mx+b
m (Slope): -0.07608
b (Y-Intercept): 0.1526
Linear Fit for: Data Set 9 | 0
y = mx+b
m (Slope): -0.08702 +/- 0.0001536
b (Y-Intercept): 0.1628 +/- 0.0001729
Correlation: -1.000
RMSE: 0.0001865
Manual Fit for: Data Set 9 | 0
y = mx+b
m (Slope): -0.09825
b (Y-Intercept): 0.1731
x(m)
0.15
0.1
0.05
0
0.5
1
1.5
t(s)
(0.321, 0.0584)

2 :
0

Time(s)	Distance in x(m) ±0.005	Ratio	ΔRatio
0.00	0.160	1.01	0.01
0.80	0.159	1.03	0.01
1.66	0.155	1.03	0.01
2.47	0.150	1.05	0.01
3.25	0.143	1.04	0.01
4.06	0.137	1.05	0.02
4.83	0.130	1.07	0.02
5.60	0.121	1.06	0.02
6.36	0.114	1.07	0.02
7.13	0.107	1.10	0.02
7.91	0.097	1.06	0.02
8.64	0.092	1.09	0.02
9.39	0.084	1.09	0.03
10.10	0.077	1.11	0.03
10.90	0.070	1.11	0.03
11.60	0.063	1.09	0.04
12.40	0.058	1.14	0.04
13.10	0.051	1.16	0.05
13.80	0.043	1.17	0.06
14.60	0.037	1.19	0.07
15.30	0.031	1.24	0.09
16.00	0.025	1.35	0.13
16.70	0.019	1.37	0.18
17.50	0.014	1.70	0.34
18.20	0.008	1.60	0.54
18.90	0.005		

7.8

Time(s)	Distance in x(m) ±0.005	Ratio	ΔRatio	Logarithmic decrement	ΔLogarithmic decrement	Damping factor	ΔDamping Factor
0.00	0.160	1.02	0.01	0.019	0.013	0.0043	0.0000
0.875	0.157	1.04	0.01	0.039	0.013	0.0088	0.0000
1.76	0.151	1.09	0.02	0.083	0.016	0.0186	0.0000
2.61	0.139	1.11	0.02	0.106	0.019	0.0239	0.0000
3.45	0.125	1.15	0.02	0.137	0.023	0.0308	0.0001
4.26	0.109	1.12	0.02	0.109	0.023	0.0246	0.0000
5.07	0.098	1.19	0.03	0.178	0.032	0.0399	0.0001
5.88	0.082	1.26	0.03	0.233	0.044	0.0524	0.0002
6.67	0.065	2.10	0.10	0.744	0.207	0.1651	0.0028
8.23	0.031	1.56	0.13	0.447	0.145	0.1001	0.0012
8.99	0.020	2.65	0.50	0.975	0.442	0.2144	0.0079
9.79	0.007	2.29	1.12	0.830	0.604	0.1836	0.0092
10.5	0.003						

15.6

Time(s)	Distance in x(m) ±0.005	Ratio	ΔRatio	Logarithmic decrement	ΔLogarithmic decrement	Damping factor	ΔDamping Factor
0.00	0.160	1.14	0.02	0.134	0.019	0.0300	0.0000
0.89	0.140	1.30	0.02	0.260	0.036	0.0583	0.0002
1.74	0.108	1.29	0.03	0.251	0.040	0.0565	0.0002
2.57	0.084	1.66	0.05	0.509	0.108	0.114	0.001
3.37	0.051	2.79	0.21	1.03	0.361	0.225	0.007
4.16	0.018	3.62	0.96	1.29	0.658	0.278	0.015
5.02	0.005						

23.4

Time(s)	Distance in x(m) ±0.005	Ratio	ΔRatio	Logarithmic decrement	ΔLogarithmic decrement	Damping factor	ΔDamping Factor
0.00	0.160	1.159	0.016	0.148	0.020	0.033	0.0001
0.858	0.138	1.438	0.025	0.363	0.056	0.081	0.0004
1.72	0.096	1.808	0.053	0.592	0.131	0.132	0.0014
2.54	0.053	5.784	0.748	1.755	0.803	0.367	0.0246
3.35	0.009	1.836	0.591	0.608	0.402	0.135	0.0045
3.63	0.005						

31.2

Time(s)	Distance in x(m) ±0.005	Ratio	ΔRatio	Logarithmic decrement	ΔLogarithmic decrement	Damping factor	ΔDamping Factor
0.00	0.160	1.22	0.085	0.200	0.0255	0.0450	0.0001
0.917	0.131	1.45	0.027	0.374	0.0593	0.0839	0.0004
1.77	0.0901	2.57	0.102	0.946	0.286	0.208	0.0050
2.59	0.035	7.00	1.667	1.946	0.992	0.401	0.0332
3.35	0.005						

39

Time(s)	Distance in x(m) ±0.005	Ratio	ΔRatio	Logarithmic decrement	ΔLogarithmic decrement	Damping factor	ΔDamping Factor
0.00	0.160	1.24	0.017	0.216	0.0276	0.049	0.0001
0.892	0.129	1.56	0.031	0.444	0.0768	0.099	0.0006
1.74	0.0827	3.52	0.193	1.26	0.461	0.272	0.010
2.56	0.024	4.70	1.188	1.55	0.785	0.329	0.022
3.49	0.005						

46.8

Time(s)	Distance in x(m) ±0.005	Ratio	ΔRatio	Logarithmic decrement	ΔLogarithmic decrement	Damping factor	ΔDamping Factor
0.00	0.160	1.27	0.018	0.239	0.031	0.0537	0.014
0.90	0.126	1.64	0.034	0.496	0.092	0.111	0.041
1.76	0.077	3.95	0.256	1.37	0.534	0.296	0.230
2.58	0.019	3.88	1.017	1.36	0.691	0.292	0.297
3.41	0.005						

54.6

Time(s)	Distance in x(m) ±0.005	Ratio	ΔRatio	Logarithmic decrement	ΔLogarithmic decrement	Damping factor	ΔDamping Factor
0.00	0.160	1.60	0.0260	0.47	0.08	0.105	0.001
0.88	0.100	6.76	0.527	1.91	0.85	0.395	0.028
1.72	0.015	2.96	0.825	1.09	0.57	0.237	0.011
2.51	0.005						

62.4

Time(s)	Distance in x(m) ±0.005	Ratio	ΔRatio	Logarithmic decrement	ΔLogarithmic decrement	Damping factor	ΔDamping Factor
0.00	0.160	1.82	0.032	0.598	0.120	0.133	0.001
0.86	0.088	8.30	0.885	2.116	0.991	0.430	0.036
1.75	0.011	2.12	0.650	0.751	0.444	0.167	0.006
2.22	0.005						